

Practical Exercises for Learning to Construct NMR/MRI Probe Circuits

DUSTIN D. WHEELER,¹ MARK S. CONRADI²

¹*Department of Chemistry, Washington University, 1 Brookings Drive, Saint Louis, MO 63130*

²*Department of Physics, Washington University, 1 Brookings Drive, Saint Louis, MO 63130*

ABSTRACT: Nine sets of laboratory exercises are designed to acquaint the student with many aspects of NMR/MRI probe resonant circuits. These include demonstrations of stray inductance and capacitance, matching a resonant circuit to $50\ \Omega$, inductive coupling, doubly resonant circuits, loading effects of tissue, and radiation from the probe circuit. Many of the exercises are for the bench, but some are performed in an NMR system. The exercises should lead to an understanding of, as well as some intuitive knowledge about, basic NMR/MRI probe circuits. © 2012 Wiley Periodicals, Inc. Concepts Magn Reson Part A 40: 1–13, 2012.

KEY WORDS: probe; tuned circuit; impedance matching; coupling; double resonance; remote tuning

INTRODUCTION

Many groups build their own NMR/MRI probes. The author's group, for example, does NMR at high pressures and extreme temperatures; there is little choice but to build these locally. There are several texts that can provide guidance, including those by Fukushima and Roeder (1) and the very thorough treatment by Mispelter et al. (2). For gaining familiarity with tuned circuits in general, texts of the "electronics for physicists" variety are not very helpful, but amateur radio

handbooks (3) remain useful for this purpose. A good discussion of the reasons for impedance matching NMR coils using tuned (or "tank") circuit approaches has appeared (4). These reasons are maximization of the radio-frequency (RF) field \mathbf{B}_1 developed by a given transmitter RF power and maximization of the RF signal power delivered to the receiving preamplifier by a given set of precessing nuclear spins. The student should read some elementary treatment about resistances and reactances in AC circuits.

However, for the student intent on building his own probe, the texts are simply not enough. The student needs to build some resonant circuits and see for himself the many effects. To that end, this article describes nine sets of exercises to give the novice probe builder practical experience. The aim is to develop both intellectual understanding and some intuition for probe circuits. The first exercises are basic and some of the later ones are a bit specialized. The student can pick and choose the ones of relevance to

Received 9 October 2011; revised 12 December 2011; accepted 22 December 2011

Correspondence to: Mark S. Conradi. E-mail: msc@wuphys.wustl.edu

Concepts in Magnetic Resonance Part A, Vol. 40A(1) 1–13 (2012)

Published online in Wiley Online Library (wileyonlinelibrary.com). DOI 10.1002/cmra.21221

© 2012 Wiley Periodicals, Inc.

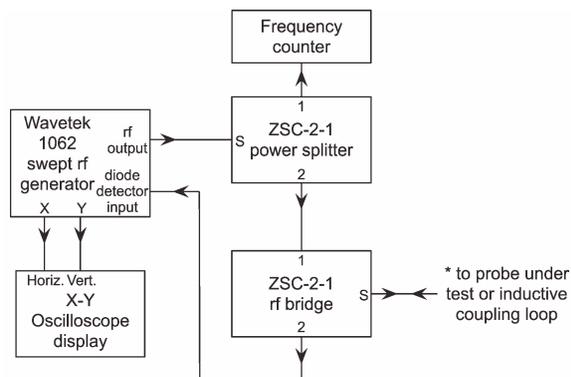


Figure 1 Reflectometer built around a coaxial bridge (Mini-Circuits ZSC-2-1) and a swept frequency RF generator. The upper splitter simply divides the generator output between the frequency counter and the RF bridge.

his own needs. But these really are intended as exercises. Like mathematics and computer programming, this is not a spectator sport.

EXPERIMENTAL METHODS

Many of the exercises are performed on the RF bench. Our apparatus is a Wavetek model 1062, 0.1–400 MHz swept RF generator with a $50\ \Omega$ reflectometer, as shown in Fig. 1. The frequency counter allows the center frequency to be measured, by disabling the sweep (CW mode). Some groups use a frequency synthesizer to serve as a “marker,” injecting its signal at the appropriate port of the Wavetek. Wavetek sweepers are no longer manufactured, but are available on the used market, including the web. The Mini-Circuits ZSC-2-1 is a coaxial $50\ \Omega$ in-phase splitter/combiner. It serves here as an RF version of the Wheatstone bridge (try looking up on the web). When the load is $50\ \Omega$ resistive, the diode detected reflection becomes zero. The second splitter is just to send a portion of the generator’s output to the frequency counter. The swept frequency aspect is a real convenience for this kind of work. Morris Instruments in Canada sells complete swept frequency reflectometer bridges that can even be placed in the stray field region of a magnet, unlike cathode-ray tube oscilloscopes. We have used and liked model 405NV+. Another option is the very compact, computer-interfaced Vector Network Analyzer model DG8SAQ from SDR-Kits in the UK (e-mail address: sdrkits@gmail.com); these are inexpensive and can perform more sophisticated measurement tasks, in addition to simple reflectometry.

A few of the exercises need to be performed in an NMR magnet with a spectrometer. Each group will

have its own way of proceeding, since there is so much variability in this equipment. If more of the exercises required an NMR spectrometer, the student would have to fight for spectrometer time, since these exercises will not lead *directly* to money or publication. So, the RF bench exercises are advantageous in this light.

In general, the exercises are intended for the student to do, but it will be helpful to have someone more experienced to look over his shoulder occasionally.

PRACTICAL EXERCISES

Resonance Frequency of Tuned Circuits

An inductor L and capacitor C resonate when the magnitudes of their impedances are equal, $\omega L = 1/\omega C$. Thus, the resonance condition is

$$\omega^2 LC = 1. \quad [1]$$

Here, ω is $2\pi f$ (where f is in Hz or cycles per second and ω is in radians per second). Single layer round solenoidal coils have inductance L (in microhenries),

$$L = \frac{0.4n^2 r^2}{9r + 10x} \quad [2]$$

where r and x are shown in Fig. 2(a) and are in cm; n is the number of turns (I). For r and x values in inches, omit the factor of 0.4.

The student should build some single layer solenoidal coils and resonate them with suitable ceramic or mica capacitors. A good place to start is 5 or 6 turns of stiff wire (18 wire gauge), with the coil about 2.5 cm in diameter and a 47 pF capacitor; this circuit will resonate near 30 MHz. The reflectometer should have a 2 cm, one-turn loop (“sniffer loop”) at the end of its coaxial cable (at * in Fig. 1); this inductive link couples to the tuned circuit by mutual inductance when the link is brought close to the tuned circuit’s coil. The resonant frequency is evident as a sharp pip of decreased reflection. The sniffer loop allows one to find the resonance frequency without any direct connections to the circuit under test, a real convenience. One should build three sniffer loops of various sizes, just to have at hand. The student can verify by varying L (or C) that the resonant frequency changes according to Eq. [1]. If the number of turns n is doubled holding r and x constant, the inductance L should quadruple and the resonant frequency ω (or f) should decrease by a factor of two.

Not just a coil, but any length of wire, has inductance (3). Take the coil and 47 pF capacitor from the above exercise and separate them by 10–20 cm, so there are now long wires in series with

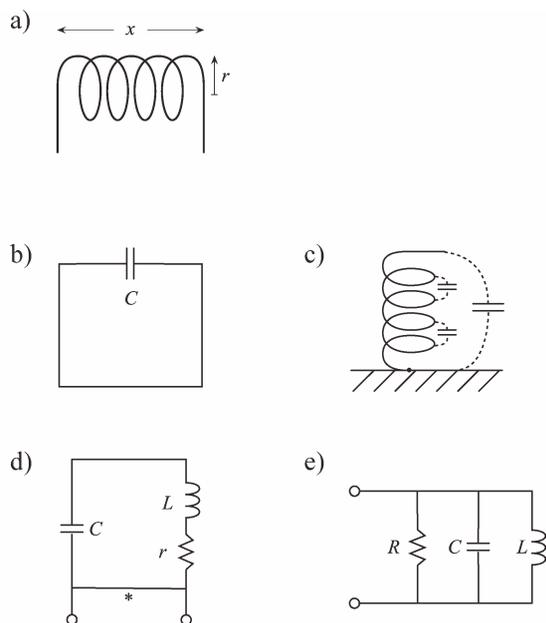


Figure 2 (a) Solenoidal coil with dimensions labeled. (b) Square surface coil made of adhesive copper tape. (c) Helical resonator in which stray capacitance resonates the coil. (d) Loss is represented by a series resistor r . (e) Loss is represented by parallel resistor R .

each side of the coil. The resonant frequency decreases due to the stray (that is, unintentional) inductance of the long leads. Next, one can eliminate the coil and keep only the long leads (joining them together where the coil used to be). This forms a tuned circuit and allows one to determine the lead inductance from Eq. [1].

Here, is another demonstration of the inductance of straight wires. Build a coil of 6 mm width adhesive copper tape (from 3M Corp., sold by Digi-Key), with $C = 120$ pF closing the circuit and resonating near 27 MHz [Fig. 2(b)]. Solder the joints where the tapes overlap; a square 10 cm on a side, built on Plexiglas sheet, works well. Stiff wire bent into a square loop also works well. By moving the sniffer loop around, it becomes clear that the LC circuit has the largest RF magnetic field near the copper tape, and much less in the center. Equation [1] can be used to calculate the inductance of the square coil; from that one can find the inductance per length of the tape conductor. The take-home message is “all lengths of wire, coiled or not, have inductance.” Incidentally, the flat coil is a surface coil; in some MRI applications, these are placed on or near the target tissue (5). The RF field extends about one radius above and below the plane of the coil. This can be verified by moving a sniffer loop above and below the flat coil.

Using a slightly larger diameter loop provides sufficient coupling to allow detection of the resonance.

Stray capacitance can also be important. A helical resonator appears in Fig. 2(c). This circuit is a coil sitting on top of a ground plane (thin copper tape or sheet will work); the circuit is closed or completed by stray capacitance between the turns and from the coil end to the ground plane (shown by dashed lines). Try a 2.5 cm diameter coil, 4 cm long with eight turns, all above a ground plane. With the sniffer loop, the resonance appears near 300 MHz.

Another demonstration of stray capacitance is the spiral or scroll coil. Wind a three-turn coil in a spiral, with Teflon film insulation between the turns, all supported on a glass tube. Use copper tape of 1.27 cm (0.5 inch) width (the adhesive kind from 3M works well) with one surface covered completely by thin Teflon sheet (0.05 mm thickness works well). Use masking tape to hold the copper and Teflon starting ends to a 10-mm diameter glass tube and tightly wind three turns, taping over it when finished, so it does not unwind. Find the resonant frequency with the sniffer loop and reflectometer; ours was near 240 MHz. This observation is meant to show the effect of capacitance between the turns. Although this is an instructive example, it is not very practical, as the turn-to-turn capacitance is reduced by small air gaps, so the resonant frequency of systems made in the authors' laboratory is not adequately stable.

Copper is a good conductor, but not perfect. There is always dissipation or loss in the tuned circuit. Making matters worse, the current travels only on the outer skin of the wire or ribbon, to a depth δ (the skin-depth; δ),

$$\delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}}, \quad [3]$$

where σ is the conductivity and μ_0 is the magnetic permeability of free space, $4\pi \times 10^{-7}$ N/A². For copper at 20 MHz, δ is about 12 μm ; at higher frequencies, δ is even smaller. Because the current carrying region is so thin, the effective resistance is higher than at DC ($\omega = 0$).

The reciprocal of the loss is expressed by Q . For L in series with a small resistance r as in Fig. 2(d), $Q = \omega L/r$. For L in parallel with a large R as in Fig. 2(e), $Q = R/\omega L$. Clearly $Q = \infty$ corresponds to an ideal, loss-free coil, either $r = 0$ or $R = \infty$. Note that r or R in Figs. 2(d,e) can represent a real physical resistor, the loss in the copper conductor, or *any other loss* in the system (in the capacitor or from radiation by the circuit or from animal tissue in or near the coil). Q is also approximately the ratio of the

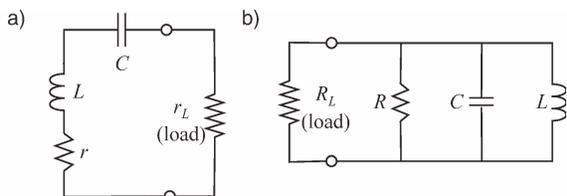


Figure 3 (a) Series-tuned circuit connected to a load, r_L . To remove the load, replace it by a short-circuit. (b) Parallel resonant connection to a load R_L . These examples demonstrate that $Q_L = \frac{1}{2} Q_U$, when impedance matched.

center (resonance) frequency to Δf of the response, the full width at half of maximum, as displayed on the reflectometer. In general, Q is defined as 2π times the ratio of the time-average stored energy in the tuned circuit to the energy lost in each cycle (3).

Try to decrease the coil Q to say 20 (compared to the approximately 100 of a good coil at 30 MHz), using a series r or parallel R , as in Fig. 2(d,e). At a given location of the sniffer loop, the response is weaker and broader. To get a large response (as it was with the high- Q coil), one must place the sniffer loop closer to the resonated coil and may be used a larger sniffer loop.

Matching the Tuned Circuit

There are many ways to match a tuned circuit to the universal standard of 50Ω resistive. Transmitters, amplifiers, and coaxial cable are all designed for this source/load impedance, so the coil's impedance should be transformed to be 50Ω resistive, too.

A typical coil might have an inductive impedance $i\omega L$ of $i100 \Omega$ and a Q of 100. So, in Fig. 2(d), the series resistance r is 1Ω . The complex impedance of the coil is $i\omega L + r = i100 + 1$. At the resonant frequency, the capacitor C is chosen to have complex impedance $-i/\omega C = -i100$. So, the series combination of L , r , and C (imagine cutting the circuit of Fig. 2(d) along the lower leg at * and connecting the spectrometer or reflectometer to the terminals shown) is the sum of these, an impedance of 1Ω resistive (at resonance). In general, series resonant circuits have impedances that are much smaller than 50Ω . Conversely, in the parallel representation of Fig. 2(e), the same coil with $Q = 100$ at the same frequency would result in the parallel R having value $R = 10 \text{ k}\Omega$. If the spectrometer or reflectometer were connected in parallel with L (or C or R), the $10 \text{ k}\Omega$ impedance (the parallel combination of $10,000$, $i100$, and $-i100$) would be a bad mismatch to the standard 50Ω system. Parallel tuned circuits generally have impedances at resonance that are much greater than 50Ω .

Clearly, *tuning* the coil removes or cancels its reactance, resulting in a real (resistive) remainder. This must be followed by or combined with some transformer-like device to transform the too-low series impedance of Fig. 2(d) [or too-large parallel impedance of Fig. 2(e)] to 50Ω .

Before proceeding to the practical exercises, it is important to note that, when the tuned circuit is impedance matched to the (50Ω) load, its Q is only half of the unloaded value (that is, when the circuit was not connected to the load). That is,

$$Q_L = \frac{1}{2} Q_U, \quad [4]$$

where Q_L and Q_U are the loaded and unloaded values of Q . To impedance match the circuit to a 50Ω resistive load, one needs to adjust the current through the load (or the voltage across the load) so the power dissipated in the external load is equal to the power dissipated in the loss of the tuned circuit itself [that is, in r or R of Figs. 2(d) or 2(e), respectively].

Expression [4] is quite general and applies (for example) to ESR cavity resonators or even acoustic resonators where there may be no equivalent lumped-element circuit representation. But, it can be made plausible (“proof by example”) using the series and parallel resonant configurations of Fig. 3. In Fig. 3(a), the 50Ω load (spectrometer) has been transformed to r_L . Clearly, the load is matched to the tuned circuit when $r_L = r$ (note that the load is transformed to match the LCr circuit, which is just a different view from the usual transformation of the LCr to match the load). In this loaded case, the Q of the series circuit is $Q_L = \omega L / (r + r_L)$ which is $Q_L = \omega L / 2r$ for $r_L = r$. If the load were removed and replaced by a short, the unloaded Q would be $Q_U = \omega L / r$. For this case $Q_L = \frac{1}{2} Q_U$, when matched. In the parallel circuit of Fig. 3(b), the unloaded Q (just remove R_L) is $Q_U = R / \omega L$. For the loaded circuit, $Q_L = (R_L \parallel R) / (\omega L)$ with $R_L \parallel R$ being the parallel combination of R_L and R . For the impedance matched case of $R_L = R$, $R_L \parallel R = R \parallel R = R/2$, so $Q_L = (R/2) / \omega L = \frac{1}{2} Q_U$, again. The relation is easiest to prove for these two simple cases, but it is correct for all cases.

For the circuit of Fig. 4(a), the coupling capacitor C_C is typically many times smaller than the tuning capacitor C_T . Suppose a voltage is induced in L from precessing nuclear spins. The *circulating current* I of the tuned circuit flows through L and then is *divided*, with most going through C_T and only a small fraction I' through C_C and the load R_L . One adjusts C_C so that the current I' through the load results in dissipation

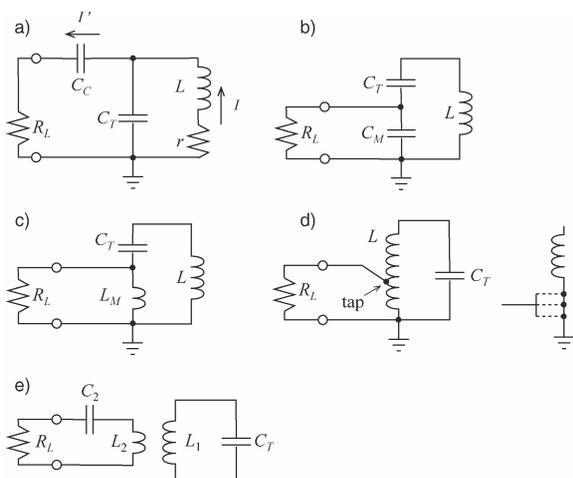


Figure 4 Schemes for matching (coupling) the coil to transform its impedance to be 50 Ω , resistive. (a) Capacitive top-coupling or current divider. (b) Capacitive voltage division. (c) Inductive voltage division and the very related (d) inductive tapping voltage division. (e) Mutual inductance coupling. In each case, the two terminals are for connection to the coaxial cable going to the spectrometer or reflectometer. The coil loss is represented by r in (a) only.

$P_{\text{ext}} = (I')^2 R_L$ which equals (if matched) the internal dissipation $P_{\text{int}} = I^2 r$. The current division factor is approximately $I'/I = C_C/(C_C + C_T)$. If C_C is too small, the circuit is undercoupled ($P_{\text{ext}} < P_{\text{int}}$); if C_C is too large, the circuit is overcoupled ($P_{\text{ext}} > P_{\text{int}}$).

Try a four-turn solenoidal inductor of 1.5 cm diameter and 1.5 cm length with C_T being a 50 pF maximum variable capacitor and C_C a 13 pF maximum variable capacitor. Trimmer capacitors are cheap and work well for bench tests, though their voltage ratings may be inadequate for use in pulsed NMR. Connect the reflectometer where R_L is drawn in Fig. 4(a).

For some value of C_C , there will be zero reflection at the resonant frequency. You should learn to distinguish undercoupling (here, C_C too small) from overcoupling (C_C too large); both situations result in a nonzero minimum reflection. That is, the reflection never gets down to the (zero reflection) line on the reflectometer display. Bringing some object, like one's hand, near the coil decreases its unloaded Q . This increases P_{int} because the hand is neither a perfect insulator nor a perfect conductor and it is coupled to the tuned circuit by stray capacitance and mutual inductance. The power dissipated internally in the tuned circuit (with the hand considered to be part of the tuned circuit) is increased. If the circuit is already undercoupled, $P_{\text{ext}} < P_{\text{int}}$, the hand's proximity increases P_{int} and makes the inequality worse; the minimum reflection becomes even larger. But, if the coil is initially overcoupled, $P_{\text{ext}} > P_{\text{int}}$, the presence of the

hand increases the internal dissipation P_{int} , eventually making the reflection at resonance pass through zero. In this way, it is easy to decide whether a mismatched tuned circuit is undercoupled or overcoupled.

In playing with the circuit of Fig. 4(a), note that the overcoupled response is broader in frequency than the response of the undercoupled case. This action shows that increasing the coupling by increasing C_C lowers the resonant frequency. As the 50 Ω load at R_L is almost always small compared to the impedance of C_C , $1/\omega C_C \gg R_L$, C_C , and C_T are effectively in parallel. By this reasoning, the resonance frequency obeys

$$\omega^2 L(C_T + C_C) = 1. \quad [5]$$

The capacitive voltage divider of Fig. 4(b) is another popular approach. Typically C_M (M for matching) is much larger than the tuning capacitor C_T . The impedance of C_M ($1/\omega C_M$) is also small compared to R_L (50 Ω). A circulating current sees C_M and C_T in series, so the resonance condition is

$$\omega^2 L(1/C_T + 1/C_M)^{-1} = 1. \quad [6]$$

As C_M is larger than C_T , the impedance of C_M is smaller than that of C_T and only a small fraction (roughly C_T/C_M) of the voltage V_L across L appears across C_M . The power dissipated in the load is $P_{\text{ext}} = (V_L(C_T/C_M))^2/R_L$. Clearly, increasing C_M reduces P_{ext} and thereby reduces the coupling (moves toward undercoupled).

Try this out, using a 50 pF variable capacitor as C_T and a variable capacitor of a few hundred pF as C_M . Note that decreasing C_M leads toward overcoupling. One practical problem with this design is that it is not easy to find variable capacitors for C_M of such large values, particularly if they must fit into a probe in a magnet.

The same principle of voltage division is at work in Fig. 4(c). Essentially, the small capacitive reactance of C_M in Fig. 4(b) is changed into a small inductive reactance in Fig. 4(c). The inductors L and L_M are effectively in series, and the resonance condition is $\omega^2 (L + L_M) C_T = 1$. Increasing L_M increases the coupling.

Our group often uses the scheme of Fig. 4(d). Here, the portion of L below the tap (the dot on L in the figure) plays the role of L_M in Fig. 4(c). Often, the inductance of the coil's ground-side lead is sufficient to serve as L_M . The wire from the load R_L is simply slid up or down the length of the ground-side lead until proper coupling is obtained, as suggested by the sketch at right in Fig. 4(d), where three

possible locations of the tap are shown. An alligator clip can speed up this process, eliminating soldering and de-soldering at each step of the trial-and-error process (beware, the clips are magnetic). Sliding the tap location down (towards ground) decreases the coupling. The Fig. 4(d) design is the RF version of an auto-transformer (3).

The approach of Fig. 4(e) is mutual inductive coupling, which has been used in earlier exercises with the sniffer loop (7, 8). As the loop-resonating capacitor C_2 is not needed, it is usually omitted at first and replaced with a short. Moving L_2 (a one- or two-turn loop) closer to the main coil L_1 increases the coupling. This is a coupling method that requires no electrical connection to the main tuned circuit; this is useful if L_1 and C_T are inside a magic-angle spinning rotor or are embedded in tissue (9), for example.

Try series resonating the loop L_2 (also known as the link) with a capacitor C_2 (find the correct C_2 in a separate test with just L_2 and C_2 , probing with a sniffer loop). One finds that optimum (matched) coupling to the main circuit (L_1 and C_T) now occurs with L_2 at a greater distance from L_1 . Decreasing the Q of the $L_1 - C_T$ circuit with a resistor shows that one must bring L_2 closer to L_1 to obtain an impedance match. Series resonating the link is handy when one cannot bring the link very close to the main coil (8).

When matched and tuned, the \mathbf{B}_1 produced by the main coil L_1 is much larger than that generated directly by the link, so the link does not perturb the RF field pattern. It is important to understand that, for the same main coil, all these methods of coupling work equally well. Finally, in many cases probes need tuning adjustments but the matching/coupling can be set to a fixed value at essentially no loss in performance. In the case of conductive samples, coupling/matching may still need adjustment. But often, a fixed coupling/matching setting works well and eliminates large, expensive, and trouble-prone adjustable capacitors. For example, C_C in Fig. 4(a) or C_M in Fig. 4(b) could be replaced with a fixed capacitor selected by trial-and-error.

A nice feature of mutual inductive (link) coupling is that there is no electrical connection between the incoming coaxial line and the main tuned circuit, unlike the designs of Figs. 4(a–d). The link coupling has, in effect, a built-in balun (2) or isolation transformer, which can be useful with large samples (animals or humans) in MRI.

Inductive Tuning

In probes at very low or very high temperatures or at extreme pressures or in exceptionally small-bore

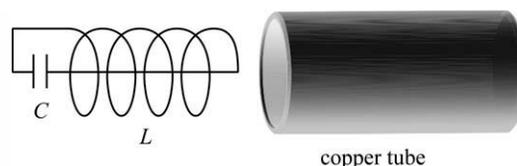


Figure 5 Inductive tuning. Sliding the copper tube (slug) closer to the coil reduces its inductance, raising the resonant frequency.

magnets, it may be inconvenient or impossible to locate an adjustable tuning capacitor close to the NMR coil. One way to deal with this is inductive tuning (Fig. 5). This is really mutual inductive tuning, so it follows nicely from the last topic, mutual inductive coupling.

Start with a four-turn solenoid with a 1.75 cm diameter and length. Locate the capacitor (47 pF works nicely here) inside the coil, to keep it out of the way. Now bring up a copper slug—a piece of copper tubing 1.5 in. (3.81 cm) long, 1.125 inch (2.86 cm) outer diameter, and 1.0 inch (2.54 cm) inner diameter; this is a standard USA water pipe size. The RF magnetic field lines from the coil neither penetrate the copper wall nor can they thread the closed conducting path. The copper rejects any RF field lines and has the effect of decreasing the inductance; simple theory shows that eddy currents (as in the copper slug) always *decrease* the inductance. As the copper slug is brought closer to the coil, the resonance frequency increases, as can be seen using a sniffer loop and the reflectometer. One can easily imagine an inductively coupled (at one end) and inductively tuned (at the other end) design, with no sliding electrical contacts or high voltage tuning adjustments at all (8). Incidentally, the RF field generated by the slug is not small, except when the frequency shift is small (<2%). Thus, the inductive tuner may perturb the RF field homogeneity of the coil.

The Effect of Tissue on Q

MRI involves doing resonant absorption on live tissue; tissue is only slightly electrically conductive, so RF fields can penetrate deeply into tissues, provided one stays below hundreds of MHz. This penetration allows MRI of humans and animals. But tissue does have some conductivity, and that causes loss and (during transmit pulses) heating of the tissue. The energy loss decreases the Q of the coil, increasing the power required to generate a given RF magnetic field \mathbf{B}_1 and decreasing the receiving spin sensitivity.

The current density \mathbf{J} in the tissue is driven by an electric field \mathbf{E} with $\mathbf{J} = \sigma\mathbf{E}$ (a version of Ohm's

law), with σ being the tissue conductivity (6). Electric fields are generated by coils two ways: (1) in a multiturn coil, there is a voltage across each turn, and the voltages add up along the several turns. There will be an electric field \mathbf{E} running along the axis of the coil. In general, \mathbf{E} is greater with more turns. (2) Maxwell's equation $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ expresses Faraday's law of induction and says that \mathbf{E} lines must circulate around \mathbf{B} -field lines whenever the \mathbf{B} is time dependent (as in NMR, at RF frequencies). This second source of \mathbf{E} is unavoidable – if an RF field \mathbf{B}_1 exists, an electric field \mathbf{E} is induced. But the first source is avoidable, as will be seen below.

Make a 2.5 cm diameter solenoidal coil with equal length and four-turns. Resonate it with a 10 pF capacitor (approximately 80 MHz). Use the reflectometer and sniffer loop to see the resonance. Putting a finger through the coil without touching it broadens the resonance because the situation has reduced the Q . The resonant frequency decreases (on the order of 0.5 MHz) because of the addition of stray capacitance to/from the finger. Record the size of these effects.

For comparison, make a single-turn coil of similar size, using copper foil or strap. To keep the frequency approximately the same, use about 16 times as much capacitance (recall Eq. [2]). Ensure the leads of the capacitor are short. The coil should be just 5% short of a full turn. Solder the capacitor across this gap with the shortest leads possible, with the capacitor body right against the coil, as in Fig. 6. Using the sniffer loop again, one finds much less broadening when inserting a finger and much less frequency shift. This situation involves primarily the unavoidable electric fields, as required by Maxwell's equation.

In general, circuits with small L and large C have smaller electric fields, resulting in less Q reduction and smaller tuning shifts from tissue (or from salt-water solvent). A famous example is the Alderman-Grant coil, where there is not only a single turn, but also divides the capacitance into several pieces along the path. Such designs are generically known as low- E coils (10).

Doubly Resonant Circuits

In molecular vibrations, coupling together two or more oscillators with the same nominal frequency results in *normal oscillating modes*, each with its own frequency. As an example, the C—O stretching vibrations of the CO₂ molecule couple together, resulting in symmetric and antisymmetric stretching modes with different frequencies. Similarly, two

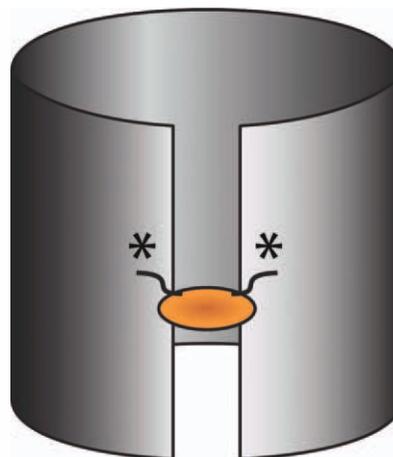


Figure 6 One-turn coil of copper foil, possibly built on a glass tube, with resonating capacitor. The capacitor is the small circle bridging the gap and is soldered at *. This is a low- E (low electric field) coil.

tuned circuits, each with the same frequency when isolated, will have two separate normal modes (at two different frequencies) when coupled. This is one method for making doubly resonant probes.

The first example appears in Fig. 7(a), where two tuned circuits are capacitively top-coupled (11). C_1 and C_2 should be about 200 pF each; L_1 and L_2 can be five-turn, 2 cm diameter and length coils. Start by fine-tuning (trimming and pruning) each tuned circuit to the same resonant frequency (ours was near 19 MHz) when the circuits are uncoupled (this means no C_C and no mutual inductive coupling; try orienting L_1 and L_2 perpendicularly, as in the letter T). One might use a small trimmer capacitor on one or both circuits, in parallel with C_1 and/or C_2 , to match the frequencies. Another method is to stretch or compress L_1 or L_2 to make small changes in the inductance, according to Eq. [2]. Connect the circuits with C_C of 47 pF, which allows one to detect a (symmetric, S) mode at the frequency of the isolated, individual LC circuits. In addition, a lower frequency (antisymmetric, AS) mode is also seen. Increasing C_C further decreases the frequency of the AS mode. Probing with the sniffer loop shows that L_1 and L_2 each participate equally in both modes, S and AS.

Connect a two-channel RF oscilloscope at y1 and y2 in Fig. 7(a); do not forget the ground connection. Use of divide-by-10 oscilloscope probes minimizes loading and detuning of the circuits. Driving one of the coils at the S-mode (higher) frequency (using the CW mode of the generator) results in voltages at y1 and y2 that are equal in magnitude and are in phase. At this frequency, there is no voltage difference across C_C so no current flows through it. If it were

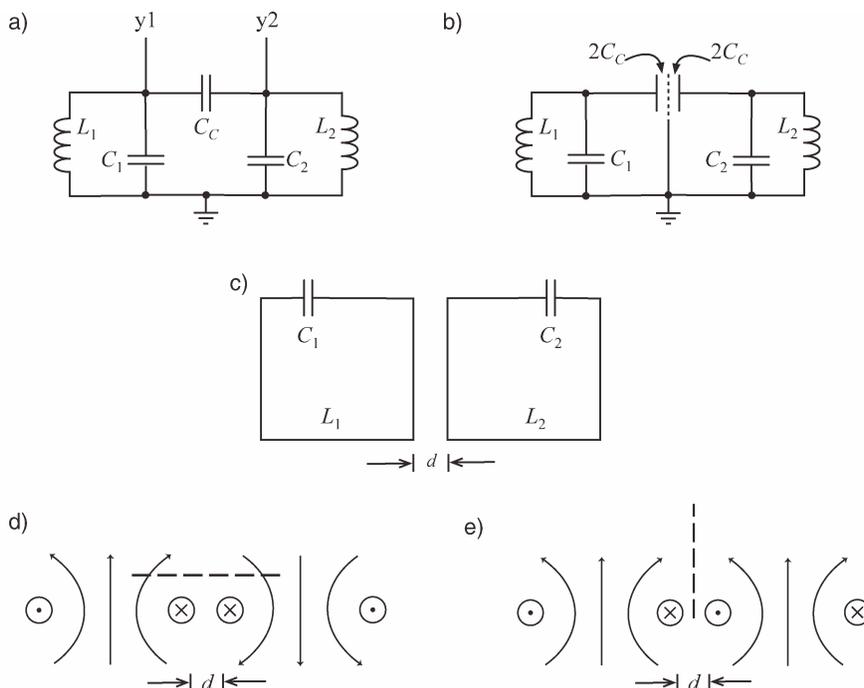


Figure 7 (a) Capacitively coupled tuned circuits. (b) Equivalent circuit for antisymmetric resonance of circuit (a). (c) Two square planar coils coupled by their mutual inductance, which varies with distance d . (d) Side view showing relative phases of currents and magnetic field lines for AS mode. (e) Same, for S mode.

removed, nothing would change. C_C does not matter to the frequency of the S mode: $\omega_S^2 L_1 C_1 = 1$ (and we note $L_1 C_1 = L_2 C_2$).

One may use the sniffer loop and the generator in CW mode at the AS (lower) frequency to generate y_1 and y_2 voltages that are equal in magnitude but out of phase by 180° . Thinking of C_C as a parallel-plate capacitor, the mid-plane of C_C has a voltage equal to the average of y_1 and y_2 , zero in the AS mode. The mid-plane of C_C could be connected to ground without changing anything, as in Fig. 7(b). The two halves of C_C are in series, so each piece has capacitance $2C_C$. The resonance frequency of the AS mode is $\omega_{AS}^2 L_1 (C_1 + 2C_C) = 1$

For NMR, L_1 might be the NMR coil and L_2 the “idler” coil. This design is especially suitable when the two desired frequencies are close to each other, like ^{19}F and ^1H or ^3He and ^1H .

Mutual inductance can also couple two resonators together and split the response into two resonant frequencies. This is presented in Fig. 7(c) for two planar and square coils (side length about 10 cm), separated by a distance d . Each can be built on a separate piece of Plexiglas sheet. Each circuit should be tuned to the same frequency when they are far apart, using the sniffer loop and reflectometer (120 pF capacitors should resonate near 27 MHz). Bring-

ing the coils close together, so distance d is nearly zero, splits the resonance into two, with one higher in frequency and one lower in frequency. For the loops as shown in Fig. 7(c), the AS mode has currents circulating in the two coils in opposite directions, so that (at a given instant) flux lines come out of the page in the middle of L_1 while they go into the page in the center of L_2 . These fluxes “add” [increasing the effective inductances; think of the RF field lines, as in the side view of Fig. 7(d)], so the frequency of the AS mode is lower than the individual, isolated resonant frequencies.

It can be shown that both coils participate in both modes. The directions of the field lines can be determined [as in Fig. 7(d)] by placing the sniffer loop straddling equally across both coils, with its plane parallel to the plane of L_1 and L_2 [as for the dashed line in Fig. 7(d)]; zero coupling occurs for the AS mode (the net flux through the sniffer loop is zero), while the S mode is evident. Move the sniffer around a bit to see the null.

In the symmetric (S) mode, the currents in the two coils circulate in the same direction, as in Fig. 7(e). The fluxes partially oppose each other, reducing the effective inductances and increasing the resonant frequency. If the sniffer loop is oriented in the vertical plane of the dashed line in Fig. 7(e), there is no

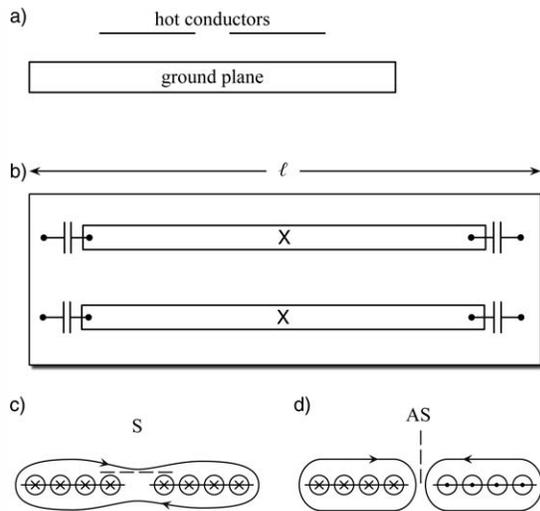


Figure 8 Two transmission-line resonators (strip-lines with a common ground plane). (a) Cross-section end view. (b) Top view. (c) Currents and RF magnetic field of S mode. (d) Same, for AS mode.

coupling to the S mode, but the AS mode is evident. The message is that one can determine what mode is observed by finding the position and orientation of the sniffer loop that results in zero coupling, and using this information to determine the field-line directions.

As d is made large, the coupling goes to zero and there is no frequency splitting. Plot the magnitude of the frequency splitting as d is varied from a large separation between the loops to a full overlap of the loops. A point of zero splitting occurs when d is about -0.1 times the length of the side of each square, a partial overlap of the coils. Drawing the flux lines makes this clear. This *overlap* “trick” is used in MRI phased arrays (12), coils with 2 or more (as many as 128!) separate small receiving surface coils. The coils are decoupled from each other by this partial overlap and then, in addition, by blocking the circulating current of each coil by means of the pre-amplifier’s matching network.

A final example of coupled resonators comes from TEM coils (13, 14). These coils are used for brain MRI at the highest frequencies, 200 MHz and above (2). They consist of a large number (~ 32) of resonant transmission line elements (strip-lines), coupled by their mutual inductance. A simple version can be built with two transmission lines coupled by mutual inductance. Besides being an exercise with coupled resonators, this is a chance to use resonators that do not look like resonators. This exercise also demonstrates how one might design circuits at very high frequencies.

Fig. 8 shows two transmission lines sharing a common ground plane. Each “hot” conductor is 20.5 cm long and 1.5 cm wide and they are separated by 0.5 cm. The ground plane is 7.5 cm wide and 25.5 cm long ($=\ell$). The “hot” conductors are 1.5 cm above the ground plane. The ground plane can be sheet or foil, but sheet is rigid and makes a more robust platform. At each end of each line is a 10 pF capacitor to ground, with *short* leads. Putting trimmer capacitors at one end allows the *individual* elements to be brought to the same frequency (near 170 MHz for us). For this adjustment, the other resonator should be disabled by temporarily removing a capacitor or by breaking the conductor at position X.

With both transmission lines functioning, there are two modes. The lower mode (155 MHz in our case) is symmetric (S), with the currents flowing parallel on the two hot conductors, as in Fig. 8(c). The magnetic field lines appear in the figure; clearly, a sniffer coil placed with its plane along the dashed line in Fig. 8(c) will not couple to this mode, because no flux penetrates the sniffer. The AS mode has the higher frequency (190 MHz) and involves antiparallel currents in the “hot” conductors, as in Fig. 8(d). For this mode, there *is* coupling for a sniffer at the above position [Fig. 8(c)], but no coupling at the position and orientation shown by the dashed line in Fig. 8(d).

A TEM coil with 24 such elements has 24 modes, just as this two-element design has two modes (2). The lowest mode (most symmetric, with all currents in-phase) and highest mode (most antisymmetric, with opposite currents in neighboring hot conductors) are nondegenerate. The other 22 modes are pairwise degenerate. The mode-pair just above the most symmetric mode has fairly uniform \mathbf{B}_1 and is good for NMR; it can be driven in quadrature to give a truly rotating RF field, for a 3 dB ($2\times$ in power) increase in \mathbf{B}_1 and in signal-to-noise.

Coupled resonators are not the only way to get a doubly resonant circuit. Consider the design of Fig. 9 and let the two coils have the same inductance. The capacitors C_1 and C_2 determine the mode frequencies. Only the case $C_1 \ll C_2$ is examined, because it is easy to analyze.

The low-frequency mode at ω_L has current flowing in series through L_1 , L_2 , and C_2 . The resonance

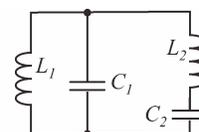


Figure 9 Simplest LC circuit offering two resonant frequencies. We consider the case of $L_1 = L_2$ with $C_2 \gg C_1$.

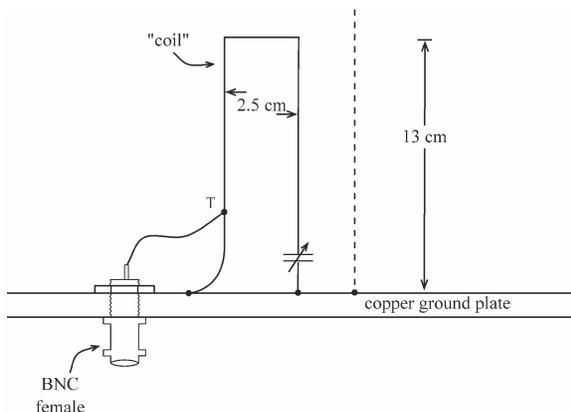


Figure 10 Very high frequency resonator that is subjected to radiation. Placing a grounded sheet next to it greatly reduces the radiation (dashed line).

condition is thus approximately $\omega_L^2 (L_1 + L_2) C_2 = 1$. At this low frequency, essentially no current flows through C_1 , because it is small and its impedance is high at the low frequency. In this mode, L_1 and L_2 have the same current and their inductances are equal by assumption, so they have the same stored (6) energy ($U = 1/2 LI^2$). Either one could be the NMR coil, with the other being the idler.

In the high-frequency mode at ω_H , capacitor C_2 (which is large) can be regarded as a short circuit. L_1 and L_2 are essentially in parallel, so the resonance condition is $\omega_H^2 C_1 (L_1 \parallel L_2) = 1$; here \parallel means “the parallel combination of.” With $L_1 = L_2$, this becomes $\omega_H^2 C_1 L_1 / 2 = 1$. As the inductors are in parallel with the same inductance, they see the same RF voltage and have the same RF current and same stored energy. Again, either L_1 or L_2 can be the NMR coil, with the idler role taken by the other.

Build two nominally equal inductors, but there is no need to match them. Solenoidal coils of four turns of 1.25 cm diameter and length will be good. Make $C_1 = 15$ pF and $C_2 = 200$ pF. Vary C_1 and C_2 to show that C_1 changes primarily the high mode and C_2 the low mode. Compare the resonant mode frequencies to the formulas above. Using the sniffer, one can show that both coils are equally involved in both modes. There are many other choices of inductances and capacitances possible for Fig. 9. Most of these do not have both coils participating equally in both modes, so neither L_1 nor L_2 would work well for NMR at **both** frequencies.

Here is another case that *does* perform well. Let L_2 be the NMR coil resonating with C_2 at $\bar{\omega}$, chosen as approximately the average of ω_L and ω_H , which are fairly close together (in percentage units). L_1 is the idler coil and is small compared to L_2 ; L_1 is

also approximately resonant with C_1 at $\bar{\omega}$. Note $L_1 \ll L_2$, so $C_1 \gg C_2$, since $L_1 C_1$ and $L_2 C_2$ are both $1/\bar{\omega}^2$. At frequencies below $\bar{\omega}$, the $L_1 C_1$ idler circuit appears inductive, leading to a resonance mode somewhere below $\bar{\omega}$ (thinking of the series circuit of L_2 , C_2 , and the inductive idler). At frequencies above $\bar{\omega}$, the $L_1 C_1$ idler appears capacitive, leading to a resonant mode above $\bar{\omega}$ (thinking of the series combination of L_2 , C_2 , and the capacitive idler). One can build such a circuit, with $L_1 = L_2/4$ and $C_1 = 4C_2$. Use the sniffer to see that L_2 is heavily involved in both resonant modes. Build it with variable capacitors and see how hard it is to tune (it is an iterative procedure). In our example, each of $L_1 C_1$ and $L_2 C_2$ was tuned to 50 MHz; the two modes appeared at 38 and 60 MHz.

Radiation

Once the linear size of the probe circuit becomes a non-negligible fraction of $\lambda/4$ (λ is the free-space wavelength, λ in meters is $300/f$, with f in MHz), the circuit radiates like an antenna (3, 6). In addition to all the other sources of loss (copper resistance and loss from conductive tissue, for example), radiation is another mechanism for losing energy from the tuned circuit. The result is a decrease in circuit Q and an increase in the required transmit power for a given B_1 RF field strength and a reduction in the received signal-to-noise.

Generally, radiation is only a problem at very high frequencies, where the wavelength λ is small, and with unshielded tuned circuits, as occasionally used in MRI. Of course, the magnet bore makes a good shield, provided the incoming coaxial cable is properly grounded to the magnet where the cable enters the magnet (small animal imaging magnets generally have an entrance plate with conducting mesh and grounded coaxial cable fittings for this purpose). Still, when tuning and testing such probes, one is often outside the shielding of the magnet. In that situation, radiation will occur, decreasing Q and leading to RF currents on the outside of the coaxial cable. These currents result in “spooky” effects, such as changes in tuning and matching when one moves near the circuit or touches the (insulated) outside of the coaxial cable. It is good to be able to recognize these effects and minimize them. Such currents on the outside of the coaxial cable can be reduced by using inductive coupling and/or baluns (2, 3).

As a demonstration, one may build a probe circuit that *invites* radiation. The coil should be stiff copper or tinned copper wire, about 28 cm long and configured as in Fig. 10, built above a ground plane of

copper or aluminum. The coil is the single loop shown and it is resonated with a 2–10 pF trimmer (like an inexpensive Sprague-Goodman FILMTRIM). Ours was set to about 7 or 8 pF, resonating near 135 MHz. One may use the sniffer loop and reflectometer to find the lowest frequency resonance. Connect the reflectometer at the BNC fitting and find the location of the tap (T in Fig. 10) for optimum matching; an alligator clip makes this adjustment easier.

Install an aluminum sheet metal *grounded* shield, according to the dashed line in the figure. This shield should be located within 3 cm of the coil and clamped or bolted in good electrical connection with the ground plane. Although it does not enclose the coil, but only stands next to it, one sees the shielding effect – the tap location for correct impedance matching is much closer to ground (less coupling required, since Q is higher) and the resonance is much narrower on the reflectometer.

It is a simple lesson. At very high frequencies, large circuits radiate and Q is reduced. By very simple shielding, this effect can generally be eliminated.

NMR Performance Tests

For this exercise, a magnet and NMR spectrometer are required. For an electromagnet, hydrogen nuclei in water usually resonate between 20 and 100 MHz. For a superconducting magnet, the fields are generally higher (4.7–9.5 T), and it is convenient to resonate the deuterons in D_2O at frequencies between 30 and 60 MHz.

Wind a solenoid with several turns and with its axis perpendicular to the static magnetic field (the solenoid can hold and support the glass sample tube) and resonate it with a tuning capacitor. Use the tap-on-the-ground-leg method of coupling from Fig. 4(d), because it is so easy. The circuit should be in a metal enclosure and rigidly located in the magnetic field, with the RF coil in the center of the homogeneous region of the magnet. Keeping the transmitter power low means that high-voltage capacitors are not needed. For narrow liquid-state lines, long RF pulses are fine. Once the circuit is tuned and matched, find the NMR signal. Record the 90° pulse duration (on-resonance) at a given transmitter power and record the initial FID amplitude. The sample should be a short, small-diameter cylinder.

Vary the size of the NMR coil holding the sample size fixed. One finds that the NMR signal after a 90° pulse is largest when the coil is just large enough to contain all the sample (high filling factor). The RF field strength B_1 is largest for the smallest coils. For really small coils, of diameters less than 1 mm, one

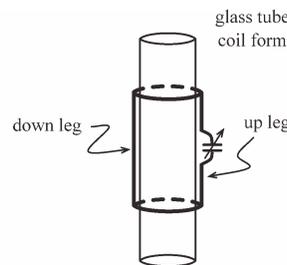


Figure 11 Sideways RF coil, following the Alderman–Grant design. The coil can be stiff wire or copper foil, possibly adhesive.

can generate remarkably large B_1 fields. Clark gives the expression (15)

$$B_1 = \frac{3}{10,000} \sqrt{\frac{PQ}{Vf}},$$

where P is transmitter power in watts, V is coil volume in cubic centimeter, and f is the resonance frequency in megahertz. The B_1 value is the (rotating) field in tesla. Clearly, large B_1 is easier to achieve at lower frequencies and in *smaller coils*. For coils of 1.0 mm diameter and length or smaller (16), very small transmitter powers generate adequate B_1 fields and a few watts can generate huge B_1 fields.

One may intentionally lower the Q of a coil with a resistor in series. Many labs have carbon film resistors, which are magnetic; the student should use the older carbon composition resistors instead, checking them for magnetism with a small magnet. Both B_1 and the received signal strength (by reciprocity; 17, 18) are expected to vary as $Q^{1/2}$.

Many NMR experiments use a vertical cylindrical sample in the vertical DC magnetic field of a superconducting magnet. In these cases, a sideways coil is necessary, one that produces B_1 perpendicular to the axis of the coil and sample.

Build a simplified version of the Alderman–Grant coil (19), as shown in Fig. 11. The coil is broken at one location and a small trimmer capacitor is used to complete the circuit. Johanson makes some small trimmers like 9710-1, 9410-1, and 9402-6, sold by Newark. With the low inductance of this one-turn design, it is easier at 200 or 300 MHz (for hydrogen). We did build one for deuterium at 31 MHz, but it required a *lot* of capacitance (1,250 pF). Inductive coupling to the spectrometer can be used. If one uses the same sample as in the previous example, the performance of the sideways coil can be compared to a solenoid of the same volume. One finds the 90° pulse time is roughly doubled for the same transmitter power; nothing seems to be as simple and perform as well as a solenoid coil.

Remote Tuning

There are occasions when it seems that one just cannot find space for suitable tuning capacitors. As mentioned earlier, inductive tuning may be a solution. Another approach is to tune and match the probe circuit with hand-selected fixed capacitors, physically near the coil, close to 50 Ω resistive and the desired frequency. Then, small tuning changes due to drift, temperature, and sample properties may be removed with a re-tuning box (remote tuner), located outside the probe where space is plentiful (20–22). The idea comes from amateur radio (3); when the antenna itself cannot be adjusted in length to be the correct impedance at all of the desired frequencies, a universal matching box is used to transform the impedance (of virtually any antenna) to 50 Ω . Of course, it must be adjusted for each frequency.

Measure 90° pulse times at a fixed transmitter power and measure the signal amplitude of the FID following a 90° pulse. Depending on the field and frequency, H₂O or D₂O may be a suitable sample. Compare the performances on transmit and receive of three arrangements: (1) the properly tuned and matched probe; (2) the probe intentionally mistuned by ± 2 and $\pm 5\%$ in frequency; (3) the probe itself intentionally mistuned by ± 2 or $\pm 5\%$, but with a retuning box located away from the probe and outside the magnet. The retuning box might be located one or two wavelengths away from the probe. Because there will be large standing waves and increased attenuation on the cable between the mistuned probe and the retuning box, the retuning solution will perform best if this cable is not too long. It seems best to make this length just long enough so the retuning box sits outside the magnet.

A paper in the literature (20) gives typical results and shows the simple circuit of the retuning box. More recently, we found at 85 MHz in a 2.0 T electromagnet that mistuning the probe by 5 MHz reduced B_1 by a factor of 2.7; with the retuning box and a cable length of one electrical wavelength, there was only a 5% reduction in B_1 (in both cases compared to the properly tuned and matched probe). Results will vary! In general, overcoupling of the probe circuit itself is important for getting a wider range of “retuning” (using the retuning box; 22).

CONCLUSIONS

The exercises presented here will not make anyone an expert in NMR probe circuits. But these exercises demonstrate the basic principles as well as a few inter-

esting special cases. You will be able to combine these concepts into useful designs for your own applications.

ACKNOWLEDGMENTS

The authors thank J. Kohlrantz and H. Lin for testing some of the circuits described here.

REFERENCES

1. Fukushima E, Roeder SBW. 1981. *Experimental Pulse NMR: A Nuts and Bolts Approach*. Reading, MA: Addison-Wesley.
2. Mispelter J, Lupu M, Briguet A. 2006. *NMR Probeheads for Biophysical and Biomedical Experiments: Theoretical Principles and Practical Guidelines*. London: Imperial College Press.
3. Danzer P ed. 1998. *The ARRL Handbook for Radio Amateurs 75th ed*. Newington, CT: American Radio Relay League.
4. Traficante DD. 1989. Impedance: What it is, and why it must be matched. *Concepts Magn Reson* 1:73–92.
5. Ackerman JJH, Grove TH, Wong GG, Gadian DG, Radda GK. 1980. Mapping of metabolites in whole animals by ³¹P NMR using surface coils. *Nature* 283:167–170.
6. Griffiths D. 1999. *Introduction to Electrodynamics*, Upper Saddle River, NJ: Prentice-Hall.
7. Froncisz W, Jesmanowicz A, Hyde JS. 1986. Inductive (flux linkage) coupling to local coils in magnetic resonance imaging and spectroscopy. *J Magn Reson* 66:135–143.
8. Kuhns PL, Lizak MJ, Lee SH, Conradi MS. 1988. Inductive coupling and tuning in NMR probes – Applications. *J Magn Reson* 78:69–76.
9. Schnall MD, Barlow C, Subramanian VH, Leigh JS. 1986. Wireless implanted magnetic resonance probes for in vivo NMR. *J Magn Reson* 68:161–167.
10. McNeill SA, Gor'kov PL, Shetty K, Brey WW, Long JR. 2009. A low-E magic angle spinning probe for biological solid state NMR at 750 MHz. *J Magn Reson* 197:135–144.
11. Haase J, Curro NJ, Slichter CP. 1998. Double resonance probes for close frequencies. *J Magn Reson* 135:273–279.
12. Roemer PB, Edelstein WA, Hayes CE, Souza SP, Mueller OM. 1990. The NMR phased array. *Magn Reson Med* 16:192–225.
13. Tropp J. 2002. Dissipation, resistance, and rational impedance matching for TEM and birdcage resonators. *Concepts Magn Reson* 15:177–188.
14. Bogdanov G, Ludwig R. 2002. Coupled microstrip line transverse electromagnetic resonator model for high-field magnetic resonance imaging. *Magn Reson Med* 47:579–593.

15. Clark WG. 1964. Pulsed nuclear resonance apparatus. *Rev Sci Instrum* 35:316–333.
16. McDowell AF, Adolphi NL. 2007. Operating nanoliter scale NMR microcoils in a 1 tesla field. *J Magn Reson* 188:74–82.
17. Hoult DI, Richards RE. 1976. The signal-to-noise ratio of the nuclear magnetic resonance experiment. *J Magn Reson* 24:71–85.
18. Hoult DI. 2000. The principle of reciprocity in signal strength calculations – a mathematical guide. *Concepts Magn Reson* 12:173–187.
19. Alderman DW, Grant DM. 1979. Efficient decoupler coil design which reduces heating in conductive samples in superconducting spectrometers. *J Magn Reson* 36:447–451.
20. Walton JH, Conradi MS. 1989. Probe tuning adjustments – need they be in the probe? *J Magn Reson* 81:623–627.
21. Rath AR. 1990. Efficient remote transmission line probe tuning. *Magn Reson Med* 13:370–377.
22. Kodibagkar VD, Conradi MS. 2000. Remote tuning of NMR probe circuits. *J Magn Reson* 144:53–57.

BIOGRAPHIES



Dustin D. Wheeler received his B.S. in chemistry at the University of Colorado — Denver. He is currently a Ph.D. student in chemistry at Washington University in St. Louis. His research interests involve studying electron-nuclear interactions in bulk and quantum confined semiconductors via laser spectroscopy and optically polarized NMR (OPNMR).



Mark S. Conradi was educated in magnetic resonance in his undergraduate and graduate years at Washington University, under R. E. Norberg. After postdoctoral study in ESR of free radicals at Oak Ridge National Laboratory under Ralph Livingston, he took an assistant professorship at the College of William and Mary. Back at Washington University since 1986, his group is involved in the study of hydrogen atom motions and site occupancies in metal-hydrides and -deuterides. The group is also using hyperpolarized gases for lung imaging in humans and animals. Sensitivity enhancement of analytical NMR is being pursued through polarization transfer from hyperpolarized spins.